HW1 Solution

5.2.1 In Example 5.2.1 the lifelength in years of a machine was known to be $X \sim \text{Exponential}(1)$, so the mode is given by 0. In Example 5.2.2 the conditional density is given by $e^{-(x-1)}$ for $x > 1$. The mode of this density is 1.

In both cases the mode is at the extreme left end of the distribution and so does not seem like a very good predictor.

5.2.2 Using the mean of a distribution to predict a future response, the mean squared error of this predictor is $E(X-1)^2 = Var(X) = 1$, where X is the future response and 1 is the mean of the distribution.

5.3.5 A single observation is from an Exponential(θ) distribution, where $\theta \in \Omega = [0, \infty)$. We can parameterize this model by the mean $1/\theta$ since the mean is a 1-1 function of θ. We can also parameterize this model by the variance, since it is a 1-1 transformation of $\theta \geq 0$. The coefficient of variation is given by $\theta^{-1}/\sqrt{\theta^{-2}} = 1$. This quantity is free of θ, and so we cannot use this quantity to parameterize the model.

5.3.7

(a) The parameter space is comprised of the possible values of θ. Hence, the parameter space is $\Omega = \{A, B\}$.

(b) The value $X = 1$ is observable only when $\theta = A$. Hence, $\theta = A$ is the true parameter. The distribution of X is

$$P(X = x) = \begin{cases} 1/2 & \text{if } x = 1 \text{ or } x = 2, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Both $\theta = A$ and $\theta = B$ are possible because $P_A(X = 2), P_B(X = 2) > 0$.

5.4.1 We have that

$$F_X(x) = \begin{cases} 0 & x < 1 \\ \frac{4}{10} & 1 \leq x < 2 \\ \frac{7}{10} & 2 \leq x < 3 \\ \frac{9}{10} & 3 \leq x < 4 \\ 1 & 4 \leq x \end{cases}, f_X(x) = \begin{cases} \frac{4}{10} & x = 1 \\ \frac{3}{10} & x = 2 \\ \frac{2}{10} & x = 3 \\ \frac{1}{10} & x = 4 \end{cases}$$

and $\mu_X = \sum_{x=1}^{4} x f_X(x) = 2, \sigma_X^2 = \left(\sum_{x=1}^{4} x^2 f_X(x)\right) - 2^2 = 1.$

5.4.7 The file extension of a file indicates the type of the file. That means the file extension is a base distinguishing the type of the file. Hence, it is a categorical variable.